

## Discrete Mathematics

### Quiz # 7 (June, 2015)

Name: \_\_\_\_\_ ID: \_\_\_\_\_

1. (30%) Answer the following questions for the poset  $(\{3, 5, 9, 15, 24, 45\}, |)$
- (a) Find the maximal and minimal elements.
  - (b) Find all upper bounds of  $\{3, 5\}$ , and figure out its least upper bound, if it exists.
  - (c) Find all lower bounds of  $\{15, 45\}$ , and figure out its greatest lower bound, if it exists.

- (a) Maximal: 24, 45, minimal: 3, 5
- (b) Upper bounds: 15, 45, least upper bound: 15
- (c) Lower bounds: 15, 3, 5, greatest lower bound: 15

2. (33%) Determine the following relations on  $\{0, 1, 2\}$  are partial orders, equivalence relations or none of them. Explain the reasons.

- (a)  $\{(0, 0), (0, 1), (0, 2), (1, 1), (2, 2)\}$
- (b)  $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 2)\}$
- (c)  $\{(0, 0), (0, 1), (1, 0), (1, 1), (2, 2)\}$

- (a) Reflexive, transitive, not symmetric but anti-symmetric, it's partial orders.
- (b) Reflexive, not transitive, neither symmetric nor antisymmetric, it's none of them.
- (c) Reflexive, transitive, symmetric, it's equivalence relation.

3. (37%)  $R = \{(1,2), (2,1), (2,3), (3,4), (4,1)\}$  and it is a relation on  $\{1,2,3,4\}$

(a) (9%) Represent the relation  $R$  by a matrix  $M_R$ .

(b) (21%) Compute  $M_R^{[2]}$ ,  $M_R^{[3]}$ ,  $M_R^{[4]}$ .

(c) (7%) Find the transitive closure of relation  $R$  on  $\{1,2,3,4\}$ .

$$(a) \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$(b) M_R^{[2]} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M_R^{[4]} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$(c) \text{ transitive closure of } R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$